

**B.C.A. Ist Semester Examination, 2023**

**MATHEMATICAL FOUNDATION OF  
COMPUTER SCIENCE**

Paper : BCA-103

Time : 2 Hours]

[Max. Marks : 70

**Note :** Attempt any five questions. All questions carry equal marks.

1. (a) Let  $A = \{1, 2, 3, 4\}$  be a set and a relation  $R$  is defined on  $A$  such that  $aRb$  if  $a \leq b$ , where  $a, b \in A$ . Check if  $R$  is :
  - (i) Reflexive
  - (ii) Symmetric
  - (iii) Transitive
  - (iv) Antisymmetric
- (b) Given  $A = \{1, 2, 3, 4\}$  and  $B = \{x, y, z\}$ . Let  $R$  be the following relation from  $A$  to  $B$  :
 
$$R = \{(1, y), (1, z), (3, y), (4, x), (4, z)\}$$
  - (i) Determine the matrix of the relation
  - (ii) Draw the arrow diagram of  $R$
  - (iii) Find the inverse relation  $R^{-1}$  of  $R$ .

(1)

2. (a) Let  $a$  and  $b$  be positive integers, and suppose  $Q$  is defined recursively as follows :

$$Q(a, b) = \begin{cases} 0 & \text{if } a < b \\ Q(a-b, b)+1 & \text{if } b \leq a \end{cases}$$

Find :

- (i)  $Q(2, 5)$
- (ii)  $Q(12, 5)$
- (b) Let  $A = \{a, b, c\}$ ,  $B = \{x, y, z\}$ ,  $C = \{r, s, t\}$ . Let  $f : A \rightarrow B$  and  $g : B \rightarrow C$  be defined by :
 
$$f = \{(a, y), (b, x), (c, y)\}$$
 and  $g = \{(x, s), (y, t), (z, r)\}$

Find :

- (i) Composition function  $g \circ f : A \rightarrow C$
- (ii)  $\text{Im}(g \circ f)$
3. (a) Write short notes on the Pano's axioms.
- (b) By using principal of Mathematical induction, prove that :

$$P(n) : 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

4. (a) Find the generating function for the fibonacci sequence defined by :

$$a_r = a_{r-1} + a_{r-2}, r \geq 2$$

with  $a_0 = 0, a_1 = 1$ .

(2)

(b) Let  $a$  and  $b$  be numeric functions such that :

$$a_r = \begin{cases} 1 & 0 \leq r \leq 3 \\ r & r \geq 4 \end{cases}$$

$$b_r = \begin{cases} 1 & 0 \leq r \leq 2 \\ r+2 & r \geq 3 \end{cases}$$

Find :

(i)  $a_r + b_r$

(ii)  $a_r b_r$

5. (a) Solve the recurrence relation :

$$a_r = a_{r-1} + 2a_{r-2}, r \geq 1$$

with the initial conditions :

$$a_0 = 0, a_1 = 1$$

(b) Solve : <https://www.sdsuonline.com>

$$a_{r+2} - 5a_{r+1} + 6a_r = 2$$

with initial condition  $a_0 = 1$  and  $a_1 = -1$ .

6. (a) By using truth table, show that :

(i)  $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

(ii)  $\sim(p \rightarrow q) \equiv p \wedge (\sim q)$

(b) Show that the proposition  $p \vee \sim(p \wedge q)$  is a tautology.

7. (a) Obtain principal disjunctive normal form for  $(p \wedge q) \vee (\sim p \wedge r) \vee (q \wedge r)$ .

(b) State and prove De-Morgan's law.

8. Write short notes on the following :

(a) Hasse diagram

(b) Equivalence relation

(c) Abelian group

(d) Cyclic group

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