S-2337

M.A./M.Sc. (IVth Semester)

Examination, 2022-23

MATHEMATICS

[Paper - I]

[Measure and Integration]

Time : 2½ Hours] **[Maximum Marks : 80**

Note : This question paper consists of two sections, Section A and B. Attempt any four questions each from section 'A' and 'B'. Limit your answers within the given answer book. B answer book will not be provided.

SECTION—A

(Short Answer Type Questions) **4×5 = 20**

1. If A and B are any two disjoint subsets R, then prove that : $m^*(A \cup B) = m^*(A) + m^*(B)$ where m^* denotes the outer measure of a set.

2. Prove that the set A is measurable if and only if its complement A^C is measurable.

3. If f is measurable function then show that | f | is also measurable but converse is not true.

4. Determine whether the function defined below is measurable :

$$f(x) = \begin{cases} x+5 & \text{if} \quad x < -1 \\ 2 & \text{if} \quad -1 \le x < 0 \\ x^2 & \text{if} \quad x \ge 0 \end{cases}$$

5. Show that the function defined on $E = [0, \infty)$ as

$$f(x) = \begin{cases} \dfrac{\sin x}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is not Lebesgue integrable on E.

6. If A and B are disjoint measurable subsets of [a, b] and if f is a bounded L-integrable function on [a, b], then prove that :

$$\int_{A \cup B} f = \int_A f + \int_B f$$

7. Prove that a monotonic function on [a, b] is of bounded variation.

8. Prove that every absolutely continuous function f defined on [a, b] is of bounded variation.

SECTION—B

(Long Answer Type Questions) **4×15 = 60**

9. Prove that union of two measurable sets is measurable.

10. Prove that Cantor ternary set is measurable and its meausre is zero.

11. Prove that a continuous function defined over a measurable set is measurable.

12. Let f and g are measurable functions defined on a measurable set E, then show that f + g, f – g and f g are measurable function over E.

13. Let f be a bounded measurable real valued function such that $a \le f(x) \le b$ on a measurable set E[p, q] < R, then prove that :

$$am(E) \le \int_E f(x)\,dx \le b\,m(E)$$

14. Prove that every bounded measurable function f defined on [a, b] is Lebesgue integrable over [a, b].

15. Prove that a function f is of bounded variation if and only if it can be expressed as difference of two monotonic functions both non-decreasing.

16. Show that a necessary and sufficient condition that a function should be an indefinite integral is that it should be absolutely continuous.

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