S-2339

M.A./M.Sc. (IVth Semester)

Examination, 2022-23

MATHEMATICS

[Paper - III]

[Linear Integral Equation]

Time : 2½ Hours] **[Maximum Marks : 80**

Note : This question paper consists of two sections, Section A and B. Attempt any four questions each from section 'A' and 'B'. Limit your answers within the given answer book. B answer book will not be provided or used.

SECTION—A

(Short Answer Type Questions) 4×5 = 20

1. Show that the function $u(x) = e^x\left(2x - \frac{2}{3}\right)$ is solution of Fredholm integral equation :

$$u(x) + 2\int_0^1 e^{x-\xi}u(\xi)d\xi = 2xe^x$$

2. Convert the following differential equation into integral equation :

$y'' + y = 0$ when $y(0) = y'(0) = 0$

3. Solve the homogeneous Fredholm integral equation.

$$g(s) = \lambda\int_0^1 e^s e^t g(t)\,dt$$

4. Solve :

$$\phi(x) = \cos x + \lambda\int_0^{\pi} \sin x\, \phi(\xi)d\xi$$

5. Show that the eigen values of a symmetric Kernel are real.

6. Find the iterated kernel for

$K(x, t) = \sin(x - 2t),\ 0 \le x \le 2\pi,\ 0 \le t \le 2\pi$

7. Find the resolvent Kernel for

$K(x, t) = x - 2t,\ 0 \le x \le 1,\ 0 \le t \le 1$

8. Define the following :

(a) Green's function (b) Resolvent Kernel

SECTION—B

(Long Answer Type Questions) 4×15 = 60

1. Transform $y'' + xy = 1,\ y(0) = 0,\ y(1) = 1$ into an integral equation.

2. Find the eigen values and the corresponding eigen functions of the integral equation

$$\phi(x) = \lambda\int_0^1 (2x\xi - 4x^2)\phi(\xi)d\xi$$

3. Solve :

$$g(s) = f(s) + \lambda \int_0^1 (s+t) g(t) dt$$

4. Using Hilbert-Schmidt theorem, find the solution of the symmetric integral equation :

$$y(x) = x^2 + 1 + \frac{3}{2} \int_{-1}^{1} (xt + x^2 t^2) y(t) dt$$

5. Solve the given integral equation by the method of successive approximations :

$$y(x) = \frac{5x}{6} + \frac{1}{2} \int_0^1 xt \, y(t) dt$$

6. Solve the given integral equation by the method of successive approximations :

$$y(x) = x - \int_0^x (x-t)\, y(t)\, dt$$

$$y_0(x) = 0$$

7. Solve the singular integral equation :

$$x = \int_0^x \frac{y(t)}{(x-t)^{1/2}} dt$$

8. Find the Green's function for the boundary value problem

$$\frac{d^2y}{dx^2} + \mu^2 y = 0$$

$$y(0) = 0, \; y(1) = 0$$

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