

S-2341

M.A./M.Sc. (IVth Semester)

Examination, 2022-23

MATHEMATICS

[Paper - IV (b)]

[Fuzzy Set Theory]

Time : 2½ Hours]

[Maximum Marks : 80

Note : This question paper consists of two sections, Section A and B. Attempt any four questions each from section 'A' and 'B'. Limit your answers within the given answer book.

B answer book will not be provided or used.

SECTION—A

(Short Answer Type Questions) 4×5 = 20

1. Write a short note on types of Fuzzy sets.
2. Prove that the sum and difference of two convex Fuzzy sets are convex.
3. Find all the α -level sets and strong α -level set of $\bar{A} = \{(x, \mu_A^{-x}) = 1 + (x - 10)^{-2}\}^{-1}$ for $\alpha = 0.3, 0.5$ & 0.7

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(1)

[P.T.O.]

4. Let $f(a)$ be a function then $f(a)$ function is defined as $-f(a) = 1 - a^w$, where $w > 0$, show that function generates the Yager class of Fuzzy complements.
5. Show that the properties of symmetry, reflexivity and transitivity are preserved under inversion for both crisp and Fuzzy relations.
6. Write a short note on value assignment in relation.
7. Prove that following proposition are tautologies :
(a) $\bar{p} \Rightarrow (p \Rightarrow q)$
(b) $(p \Rightarrow q) \Rightarrow [\{p \vee (q \wedge r)\} \Leftrightarrow \{q \wedge (p \vee r)\}]$
8. Write short note on Fuzzy propositions and linguistic variables.

SECTION—B

(Long Answer Type Questions) 4×15 = 60

9. Consider the Fuzzy sets A, B and C defined on $[0, 10]$ of real members by the membership grade functions :

$$A(x) = \frac{x}{x+2}, B(x) = 2^{-x}, c(x) = \frac{1}{1+10(x-2)^2}$$

then calculate :

- (a) $A \cup B \cup C$
- (b) $A \cap B \cap C$
- (c) $\overline{B \cap C}$

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(2)

10. Let f be a decreasing generator. Then a function g defined by $g(a) = f(0) - f(a)$ for any $a \in [0, 1]$ is an increasing generator with $g(1) = f(0)$, and its pseudo-inverse $g^{(-1)}$ is given by :

$$g^{(-1)}(a) = f^{(-1)}(f(0) - a) \text{ for any } a \in R$$

11. Let A and B be two Fuzzy numbers whose membership function are given by :

$$A(x) = \begin{cases} (x+2)/2 & \text{for } -2 < x \leq 0 \\ (2-x)/2 & \text{for } 0 < x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

$$B(x) = \begin{cases} (x-2)/2 & \text{for } 2 < x \leq 4 \\ (6-x)/2 & \text{for } 4 < x \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

Find the Fuzzy numbers $A - B$, $(A \cdot B)$ and (A/B) .

12. Let A , B be two fuzzy numbers, defined as follows :

$$A = \frac{0.2}{[0,1]} + \frac{0.6}{[1,2]} + \frac{0.8}{[2,3]} + \frac{0.9}{[3,4]} + \frac{1.0}{4} + \frac{0.5}{(4,5)} + \frac{0.1}{(5,6]}$$

$$B = \frac{0.1}{[0,1]} + \frac{0.2}{[1,2]} + \frac{0.6}{[2,3]} + \frac{0.7}{[3,4]} + \frac{0.8}{[4,5]} + \frac{0.9}{[5,6]} + \frac{1.0}{6} + \frac{0.5}{(6,7]} + \frac{0.4}{(7,8]} + \frac{0.2}{(8,9]} + \frac{0.1}{(9,10]}$$

Find the solution of equation $A + X = B$.

$$13. \text{ Let } P = \begin{bmatrix} 0.3 & 0.5 & 0.8 \\ 0.0 & 0.7 & 1.0 \\ 0.4 & 0.6 & 0.5 \end{bmatrix}, Q = \begin{bmatrix} 0.9 & 0.5 & 0.7 & 0.7 \\ 0.3 & 0.2 & 0.0 & 0.9 \\ 1.0 & 0.0 & 0.5 & 0.5 \end{bmatrix}$$

Find POQ where \circ is a max-min composition.

14. Find transitive closure $R_T(X, X)$ for the Fuzzy relation $R(X, X)$ defined by the membership matrix :

$$R = \begin{bmatrix} 0.7 & 0.5 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \\ 0.0 & 0.4 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.8 & 0.0 \end{bmatrix}$$

$$15. \text{ Given that } Q = \begin{bmatrix} 0.9 & 0.6 & 1.0 \\ 0.8 & 0.8 & 0.5 \\ 0.6 & 0.4 & 0.6 \end{bmatrix} \text{ and } r = [0.6, 0.6, 0.5] \text{ solve}$$

the Fuzzy relation equation :

$P \circ Q = r$, by using max-min composition.

16. In reference to car speed, we have the following linguistic variables :

$$A = \text{"Fast"} = \frac{0.0}{0} + \frac{0.1}{10} + \frac{0.2}{20} + \frac{0.3}{30} + \frac{0.4}{40} + \frac{0.5}{50} + \frac{0.6}{60} + \frac{0.7}{70} + \frac{0.8}{80} + \frac{0.9}{90} + \frac{1.0}{100}$$

$$B = \text{"Slow"} = \frac{1.0}{0} + \frac{0.9}{10} + \frac{0.8}{20} + \frac{0.7}{30} + \frac{0.6}{40} + \frac{0.5}{50} + \frac{0.4}{60} \\ + \frac{0.3}{70} + \frac{0.2}{80} + \frac{0.1}{90} + \frac{0.0}{100}$$

using these terms, find the membership function for the

following linguistic terms :

- (a) Very fast
- (b) Very-very fast
- (c) Fairly fast (= (fast)^{2/3})
- (d) Not very slow and not very fast
- (e) Slow or not very slow
