SDS-4255

M.Sc. (Fourth Semester) Examination, 2022

MATHEMATICS

[Linear Integral Equations]

Time - 3:00 Hours] **[Maximum Marks : 80**

Note : This question paper consists of two sections, Section 'A' and 'B'. Attempt any four questions each from Section 'A' and 'B'. Limit your answer within the given answer book. B answer book will not be provided or used.

SECTION—A

(Short Answer Type Questions) **4×5 = 20**

1. Show that the function $u(x) = (1 + x^2)^{-3/2}$ is a solution of the Volterra integral equation :

$$u(x) = \frac{1}{1+x^2} - \int_0^x \frac{\xi}{1+x^2} u(\xi) d\xi$$

2. Convert the following differential equation in to the integral equation $y'' + y = 0$ when $y(0) = y'(0) = 0$.

3. Show that the homogeneous integral equation :

$\phi(x) - \lambda \int_0^1 (3x - 2)t \phi(t) dt = 0$ has no eigenvalue and eigenfunction.

4. Find the iterated Kernels for the following kernels :

$K(x, t) = \sin(x - 2t),\ 0 \le x \le 2\pi,\ 0 \le t \le 2\pi$

5. Define Fredholm integral equation and its kind.

6. Solve :

$$y(x) = \sin x + 2\int_0^x e^{x-t} y(t) dt$$

7. Define the Green's functions.

8. Using Fredholm determinants, find the resolvent kernel, when $K(x, t) = xe^t$, $a = 0$, $b = 1$.

SECTION—B

(Long Answer Type Questions) **4×15=60**

9. Prove that the eigenvalues of a symmetric kernel are real.

10. (a) Find the eigenvalues and corresponding eigenfunctions of the homogeneous integral equation

$$u(x) = \lambda \int_0^1 \sin \pi x . \cos \pi t u(t) dt$$

(b) Solve :

$$\phi(x) = \cos x + \lambda \int_0^x \sin x . \phi(\xi) d\xi$$

11. If $R(x, t; \lambda)$ be the resolvent (or reciprocal) kernel of a volterra integral equation :

$$y(x) = f(x) + \lambda \int_a^x k(x,t) y(t) dt$$

Then $R(x,t;\lambda) = k(x,t) + \lambda \int_t^x k(x,z)\, R(z,t;\lambda) dz$.

12. Find the resolvent kernel of the Volterra integral equation with the kernel :

(a) $k(x, t) = \dfrac{2 + \cos x}{2 + \cos t}$

(b) $k(x, t) = e^{x-1}$

13. Using Hilbert-Schmidt theorem, find the solution of the symmetric integral equation :

$$y(x) = x^2 + 1 + \frac{3}{2} \int_{-1}^{1} (xt + x^2 t^2) \cdot y(t) dt$$

14. Solve the singular integral equation :

(a) $f(x) = \int_a^x \dfrac{y(t)dt}{(\cos t - \cos x)^{1/2}}, 0 \le a < x < b < \pi$

(b) $f(x) = \int_a^x \dfrac{y(t)dt}{(x^2 - t^2)^{\alpha}}, \ 0 < \alpha < 1; \ a < x < b$

15. Find the Green's function of the boundary value problem :

$y'' = 0, y(0) = y(l) = 0$

16. Solve the following integral equation :

(a) $y(x) = \cos x - x - 2 + \int_0^x (t - x) y(t) dt$

(b) $y(x) = x + \int_0^{1/2} y(t) dt$

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