SDS-4253

M.Sc. (Fourth Semester) Examination, 2022

MATHEMATICS

[Paper - I]

[Measure and Integration]

Time - 2:30 Hours] **[Maximum Marks : 80**

Note : This question paper consists of two sections, Section A and B. Attempt any four questions each from Section A and B. Limit your answer within the given answer book. B answer book will not be provided or used.

SECTION—A

(Short Answer Type Questions) **4×5 = 20**

Note : Attempt any four questions.

1. If E is a countable set, then prove that $m^*(E) = 0$.

2. If A and B are two sets with $A \subset B$ then $m^*(A) \leq m^*(B)$.

3. Show that a constant function with a measurable domain is measurable.

4. A continuous function defined on a measurable set is measurable.

5. Let $E \subset [0, 1[$ be a measurable set and $y \in [0, 1[$ be given. Then the set $E \dotplus y$ is measurable and $m(E \dotplus y) = m(E)$ where ($\dotplus$ represent sum modulo 1)

6. If A and B are disjoint measurable subsets of [a, b] and if f is a bounded L-integrable function on [a,b] then $\int_{A \cup B} f = \int_A f + \int_B f$

7. If f is a bounded function defined on [a, b] and f is R-integrable, on [a, b] then f is also L-integrable on [a, b] and hence prove that

   $L\int_a^b f = R\int_a^b f.$

8. Define Cantor's ternary set.

SECTION—B

(Long Answer Type Questions) 4×15 = 60

Note : Attempt any four questions.

9 Prove that The Outer measure of an interval is its length.

10. (a) For any sequence of sets {E_i}. Prove that

Prove that $m^*\left(\bigcup_{i=1}^{\infty} E_i\right) \le \sum_{i=1}^{\infty} m^*(E_i)$

(b) Show that union of two measurable sets is also measurable.

11. Define measurable function and show that the following statement are equivalent :

(a) $E(f > \alpha)$ is measurable for all $\alpha \in R$

(b) $E(f \ge \alpha)$ is measurable for all $\alpha \in R$

(c) $E(f < \alpha)$ is measurable for all $\alpha \in R$

(d) $E(f \le \alpha)$ is measurable for all $\alpha \in R$

12. Define :

(a) Characteristic function

(b) Step function

(c) Simple function

(d) Measurable set

13. State and prove Fatou's lemma.

14. Let f be an increasing real valued function defined on [a, b], then f is differentiable a.e and the derivative f' is measurable and

$\int_a^b f'(x)dx \le f(b) - f(a)$

15. Let f and g be bounded measurable function defined on a set E of finite measure then show that :

(a) $\int_E af = a\int_E f$, for all real number a.

(a) $\int_E (f+g) + \int_E f + \int_E g$

16. State and prove Jordan Decomposition Theorem.

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